

Dynamic Financial Analysis

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Dynamic Financial Analysis

- General ideas
- DFA and solvency testing
- Identifying sources of stochastic behaviour
- Strengths, weaknesses and limitations of DFA
- DFA in action

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Idea

For analyzing the financial effects of different strategies for insurance companies over a given time horizon there are two primary techniques in use today:

- *Scenario testing* projects results under specific scenarios in the future. The disadvantage of this deterministic approach is the fact that only a few arbitrary scenarios are tested in order to decide how good a strategy is.
- Stochastic simulation, better known as *Dynamic Financial Analysis (DFA)*. Here many different scenarios are generated stochastically with the aim of giving information about the distribution of some important variables, like surplus or loss ratio.

Fixing the Time Period

- We would like to model over as long a time period as possible in order to see the long-term effects of a chosen strategy.
- Simulated values get more and more unreliable the longer this time period is.
- A compromise must be made in order to fix the length of the simulated time period.

Which Risks Should be Modelled?

- Asset risk:
 - How will assets develop?
- Liability risk:
 - Which liabilities will be incurred?
 - When will they be incurred?
 - How big are they?
- Interrelation between both sides:
 - How do these risks depend on each other?
- It is neither possible nor appropriate to model all sources of risk: It can be dangerous to place confidence in a detailed, but perhaps inappropriate model. It is often better to use a simple model that captures only the key features.

What Does DFA Stand for?

- *Dynamic* means stochastic or variable, as opposed to static or fixed.
- *Financial* reflects the fact that not only the underwriting business is simulated but rather the total of all assets and liabilities.
- *Analysis* is defined as an examination of the whole complex, its elements and their interrelationships.

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Applications of DFA Models

Before using a DFA model, management has to choose a financial or economic measure which should be analyzed.

The most common concept is the *efficient frontier concept*:

1. Choose a measure for performance, e.g. expected surplus.
2. Choose a measure for risk, e.g.
 - ruin probability,
 - quantiles (VaR) of distribution of surplus,
 - conditional expected loss.
3. Compare different strategies by plotting the measured risk and the measured performance.

Aim of DFA

DFA gives the opportunity to compare the effects of different strategies before applying them to reality.

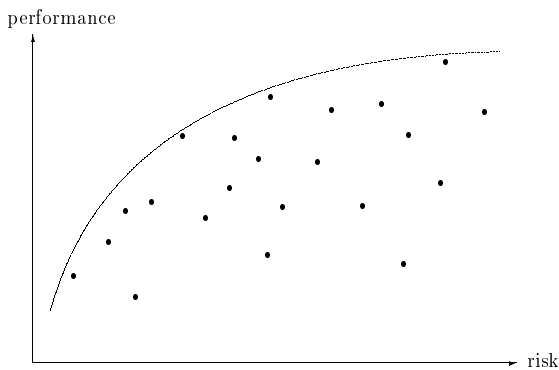
It does not necessarily give an optimal solution but leaves the decision of selecting a strategy to management.

So DFA serves as a decision tool that requires a good understanding of insurance business and some analytical/actuarial skills to be successfully implemented.

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Comparing Strategies with Respect to Performance and Risk



Efficient Frontier

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Link Between DFA and Solvency Testing

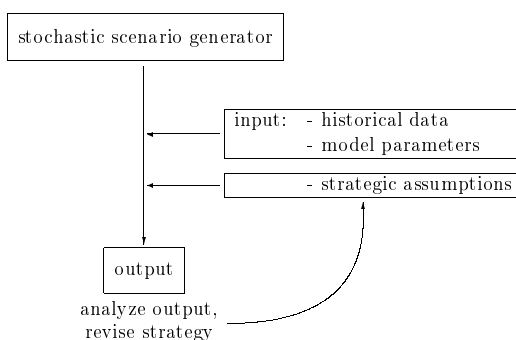
A better known concept than DFA is *solvency testing*, which deals with one central question:

Does the company have enough capital compared to the level of risk to which it is exposed, i.e. does the company have enough capital to keep the probability of ruin below a given level for the risks taken?

DFA gives us an estimate for the distribution of the surplus. A negative surplus is equivalent to the company becoming insolvent. Therefore DFA can also help answer the question of survival/ruin that is asked in solvency testing.

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Main Structure of a DFA Model



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Which Variables are Generated Stochastically?

An important step in the process of building an appropriate model is to identify the most important variables, and the sources of stochastic behaviour.

There are many possible ways of doing this.

A reasonable approach is the one implemented in Dynamo: Several different risk categories are selected and each is modelled with the help of a stochastic generator.

- Non-catastrophe losses
- Catastrophes
- Interest rates
- Stock returns
- Business cycles
- Payment patterns

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Non-Catastrophe Losses for Each LOB

Aging phenomenon: The loss ratio – i.e. the ratio of losses divided by earned premiums – decreases when the age of policy increases. Therefore it might prove useful to divide insurance business into three classes, as done in Dynamo:

- New business (superscript 0)
- Renewal business – first annual (superscript 1)
- Renewal business – second annual and subsequent (superscript 2)

For every class we can simulate

- Number of losses ($j = 0, 1, 2$)

$$N_t^j \sim \text{NB, Pois, Bin, } \dots$$

- Mean severity $X_t^j = \frac{\sum_{i=1}^{N_t^j} X_t^j(i)}{N_t^j}$

$$X_t^j \sim \text{Gamma, GPD, } \dots$$

- Losses in year t

$$\sum_{j=0}^2 N_t^j X_t^j$$

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Catastrophes

- Number of catastrophes

$$N_t \sim \text{NB, Pois, Bin, } \dots$$

$$N_1, N_2, \dots \text{ i.i.d.}$$

- Severity of an individual catastrophe $i = 1, \dots, N_t$.

$$X_t(i) \sim \text{lognormal, Pareto, GPD, } \dots$$

$$X_t(1), \dots, X_t(N_t) \text{ i.i.d.}$$

- Total severity is divided up among LOBs affected by event.

$$X_{t,k}(i) = a_{t,k}(i) X_t(i), \quad k = 1, \dots, l,$$

$$l = \# \text{ LOBs,}$$

$$\sum_{k=1}^l a_{t,k}(i) = 1.$$

- Catastrophe losses in year t

$$\sum_{k=1}^l b_{t,k} \left(\sum_{i=1}^{N_t} X_{t,k}(i) \right),$$

$b_{t,k}$ = market share of the company.

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Interest Rate Generator

- Interest rates r_t (financial assets)

$$\text{CIR: } dr_t = a(b - r_t) dt + s\sqrt{r_t} dZ_t,$$

Z_t = a standard Brownian motion.

Yearly discretization:

- $r_t = r_{t-1} + a(b - r_{t-1}) + s\sqrt{|r_{t-1}|} Z_t,$
- $r_t = r_{t-1} + a(b - r_{t-1}) + s\sqrt{r_{t-1}^+} Z_t,$
- $r_t = (r_{t-1} + a(b - r_{t-1}) + s\sqrt{r_{t-1}} Z_t)^+,$

$$Z_t \sim \mathcal{N}(0, 1), \quad Z_1, Z_2, \dots \text{ i.i.d.}$$

- Long term interest rates $R_{t,T}$

$$R_{t,T} = \frac{r_t B_T - \ln A_T}{T},$$

where

$$A_T = \left(\frac{2G e^{(a+G)T/2}}{(a+G)(e^{GT} - 1) + 2G} \right)^{2ab/s^2},$$

$$B_T = \frac{2(e^{GT} - 1)}{(a+G)(e^{GT} - 1) + 2G},$$

$$G = \sqrt{a^2 + 2s^2}.$$

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- Return on stock portfolio r_t^S

CAPM:

$$\mathbb{E}[r_t^S | R_{t,1}] = (e^{R_{t,1}} - 1) + \beta_t (\mathbb{E}[r_t^M | R_{t,1}] - (e^{R_{t,1}} - 1)),$$

where

$$\mathbb{E}[r_t^M | R_{t,1}] = a^M + b^M (e^{R_{t,1}} - 1),$$

$$\beta_t = \frac{\text{Cov}(r_t^S, r_t^M)}{\text{var}(r_t^M)},$$

$$e^{R_{t,1}} - 1 = \text{risk-free return.}$$

Assuming a lognormal distribution for $1 + r_t^S$ leads to

$$1 + r_t^S \sim \text{lognormal}(\mu_t, \sigma^2),$$

with μ_t chosen to yield

$$m_t = e^{\mu_t + \frac{\sigma^2}{2}},$$

where

$$m_t = 1 + \mathbb{E}[r_t^S | R_{t,1}],$$

σ^2 = estimated variance of logarithmic historical values.

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- Inflation i_t (loss payments)

$$i_t = a^I + b^I r_t + \sigma^I \epsilon_t^I,$$

$$\epsilon_t^I \sim \mathcal{N}(0, 1), \epsilon_1^I, \epsilon_2^I, \dots \text{ i.i.d.}$$

- Impact of i_t on each LOB

- Impact on mean number of losses:

A reasonable model is

$$\mathbb{E}[N_t^j] = (1 + \delta_t^N) \mathbb{E}[N_{t-1}^j],$$

$$\text{var}[N_t^j] = (1 + \delta_t^N)^2 \text{var}[N_{t-1}^j],$$

where

$$\delta_t^N = \max(a^N + b^N i_t + \sigma^N \epsilon_t^N, -1),$$

$$\epsilon_t^N \sim \mathcal{N}(0, 1), \epsilon_1^N, \epsilon_2^N, \dots \text{ i.i.d.}$$

- Impact on mean loss severity:

A reasonable model is

$$\mathbb{E}[X_t^j] = (1 + \delta_t^X) \mathbb{E}[X_{t-1}^j],$$

$$\text{var}[X_t^j] = \frac{(1 + \delta_t^X)^2}{1 + \delta_t^N} \text{var}[X_{t-1}^j],$$

$$\mathbb{E}[X_t(i)] = (1 + \delta_t^X) \mathbb{E}[X_{t-1}(i)],$$

$$\text{var}[X_t(i)] = (1 + \delta_t^X)^2 \text{var}[X_{t-1}(i)],$$

where

$$\delta_t^X = \max(a^X + b^X i_t + \sigma^X \epsilon_t^X, -1),$$

$$\epsilon_t^X \sim \mathcal{N}(0, 1), \epsilon_1^X, \epsilon_2^X, \dots \text{ i.i.d.}$$

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Business Cycles by LOB

Is there strong competition among insurance companies in this LOB? Is there a general recession?

We can use a homogeneous Markov chain model where we classify each LOB for every year into one of the following states

- 1 Weak competition
- 2 Average competition
- 3 Strong competition

When the company writes l LOBs, there are 3^l states of the world. Because business cycles of different LOBs are strongly correlated, only few of the 3^l states are attainable. So we have to model $L \ll 3^l$ states.

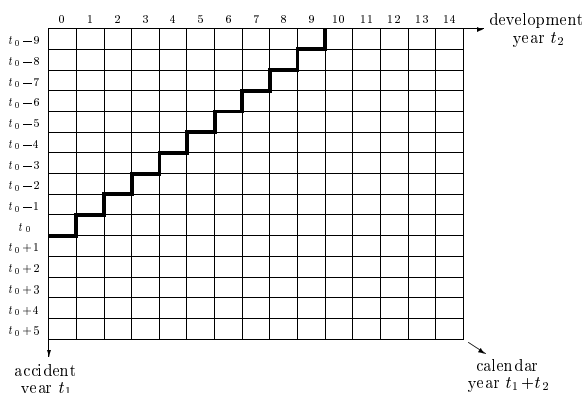
Transition probabilities p_{ij} , $i, j \in \{1, \dots, L\}$ from one year to the next are equal for every year. (Markov chain is homogeneous.)

Main effect of business cycles: The weaker the competition, the higher the premiums.

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Payment patterns

When are losses paid?



Paid losses in the upper triangle bounded by the solid line are known, while those in the lower triangle must be simulated.

To model percentages of paid losses we can use for example beta distributions.

Critical Appraisal of DFA:

Strengths of DFA

Compared to scenario testing where only a few arbitrary and possibly unrepresentative scenarios are considered, DFA gives better information on the effects of chosen strategies, because DFA simulates dynamically many different scenarios.

Because of the large number of simulations a DFA model can run, it gives us information not only on behaviour under ordinary circumstances, but also when extremal events occur. Of course the stochastic generators must be sufficiently flexible to generate occasional extreme values.

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Weaknesses of DFA

Because reality is complex, it's not possible to model all sources of risk. We have to restrict attention to some key risk factors. So in a DFA model there is not only the randomness by reason of the inherent variability, but also the uncertainty caused by incomplete knowledge.

Generally DFA overestimates probability of ruin since it does not take into consideration that an insurance company has the opportunity to make additional capital available – e.g. by issuing stocks – when it runs the risk of ruin.

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Limitations of DFA

DFA does not provide an optimal strategy. It serves as a decision tool that helps management compare different strategies. When a DFA model is used without enough actuarial knowledge, it is only a black box of limited utility.

Because reality can never be represented perfectly, we should of course always be cautious, and never rely completely upon the output produced by a DFA model.

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DFA in Action

Model assumptions are:

- Time horizon: 10 years.
- Performance measure: expected surplus.
- Risk measure: ruin probability.
- Only 1 LOB.
- New business and renewal business are not modelled separately.
- Number of non-catastrophe losses \sim NB (154, 0.025).
- Mean severity of non-catastrophe losses \sim Gamma (9.091, 242), inflation-adjusted.
- Number of catastrophes \sim Pois (18).
- Severity of individual catastrophes \sim lognormal (13, 1.5^2), inflation-adjusted.
- Market share: 5%.
- Written premiums in the last year: 20 million.
- Expenses: 28.5% of written premiums.
- Optional excess of loss reinsurance with deductible 500 000 (inflation-adjusted), and cover ∞ .
- Premiums for reinsurance: 175 000 p.a. (inflation-adjusted).

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- For interest rates we use the discretization $r_t = r_{t-1} + a(b - r_{t-1}) + s\sqrt{|r_{t-1}|}Z_t$.
- Parameters for interest rate generator: $a = 0.25$, $b = 5\%$, $s = 0.1$, $r_1 = 2\%$.
- Parameters for generating return on stock portfolio: $a^M = 4\%$, $b^M = 0.5$, $\beta_t \equiv 0.5$, $\sigma = 0.15$.
- Parameters for modelling inflation: $a^I = 0\%$, $b^I = 0.75$, $\sigma^I = 0.025$.
- No impact of inflation on the number of claims for the modelled LOB.
- Parameters for modelling the impact of inflation on the severity of claims for the modelled LOB: $a^X = 3.5\%$, $b^X = 0.5$, $\sigma^X = 0.02$.
- Business cycles: 1 = weak, 2 = average, 3 = strong. State in year 0: 1 (weak).
Transition probabilities:
 $p_{11} = 60\%$, $p_{12} = 25\%$, $p_{13} = 15\%$,
 $p_{21} = 25\%$, $p_{22} = 55\%$, $p_{23} = 20\%$,
 $p_{31} = 10\%$, $p_{32} = 25\%$, $p_{33} = 65\%$.
- Payment patterns are deterministic.

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- All liquidity is reinvested. There are only two investment possibilities:
 - 1) buy a risk-free bond with maturity one year,
 - 2) buy an equity portfolio with a fixed beta.
- Market valuation: assets and liabilities are stated at market value, i.e. assets are stated at their current market values, liabilities are discounted at the appropriate term spot rate determined by the model.
- No transaction costs.
- No taxes.
- No dividends paid.
- Initial surplus: 12 million.

In this model one can choose:

- How many simulations should be run.
- Whether reinsurance should be purchased or not.
- How the liquidity is divided between bond and portfolio.

Example with 10 000 Runs

Expected surplus & ruin probabilities for twelve different strategies:

	with reinsurance	without reinsurance
100 % bonds 0 % stocks	23.33 mio. 0.50 %	23.42 mio. 1.01 %
50 % bonds 50 % stocks	25.17 mio. 2.07 %	25.38 mio. 2.70 %
0 % bonds 100 % stocks	27.34 mio. 9.82 %	27.41 mio. 10.26 %
≤ 5 mio. bonds rest stocks	26.83 mio. 6.05 %	27.13 mio. 6.39 %
≤ 10 mio. bonds rest stocks	26.25 mio. 3.83 %	26.40 mio. 4.00 %
≤ 20 mio. bonds rest stocks	24.60 mio. 0.79 %	24.66 mio. 1.52 %

